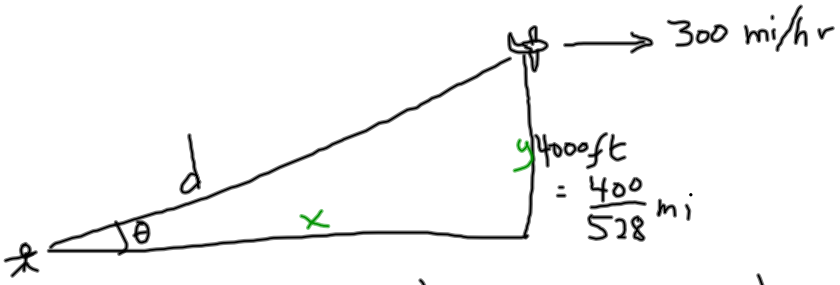
24b)



How fast is distance d changing at the instant that $\theta = 30°$

Know	≈Knowable	Need to Know
$\theta = 30°$	from 30-60-90 special Δ, x, d at instant	$\frac{dd}{dt}$
$\frac{dx}{dt} = 300$ mph		
$y = \frac{400}{528}$ mi		

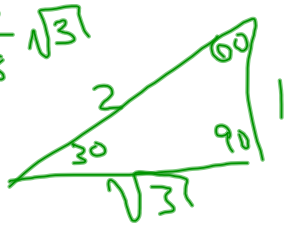
1. IV
2. $x^2 + y^2 = d^2$
3. $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = d\frac{dd}{dt}$

$\frac{400}{528}\sqrt{3}$ · 300 + $\frac{400}{528}$ · 0 = $\frac{800}{528}$ · ?

b/c constant y

$\frac{400}{528}\sqrt{3}$



$\left(\frac{400}{528}\sqrt{3}\right)(300) = \frac{800}{528}\frac{dd}{dt}$

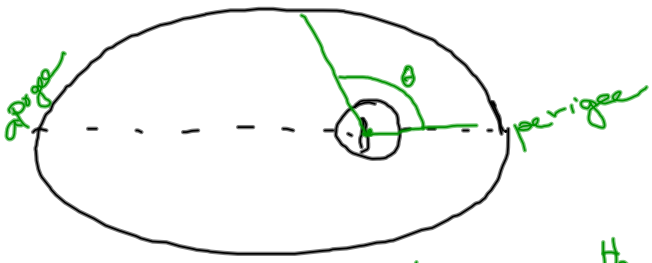
$150\sqrt{3} = \frac{dd}{dt}$

$150\sqrt{3}$ mi/hr

$\frac{mi}{hr} \Rightarrow$ # $\frac{ft}{mi}\frac{hr}{s}$

$150\sqrt{3}\ \frac{mi}{hr} \cdot \frac{5280}{1} \cdot \frac{1}{3600}$ ft/sec

23) $r = \dfrac{4995}{1+0.12\cos\theta}$



a) find altitude
- plug 0 in for θ
- plug π in for θ
- subtract 3960 mi

radius of earth = 3960 mi

b) At the instant when $\theta = 120^\circ \left(\frac{2\pi}{3}\text{ rad}\right)$

$$\frac{d\theta}{dt} = 2.7^\circ/\text{min} \cdot \frac{\pi}{180}$$

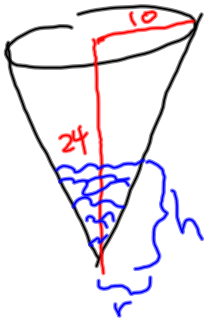
$$r = 4995(1+0.12\cos\theta)^{-1}$$

$$\frac{dr}{dt} = 4995\left(-(1+0.12\cos\theta)^{-2}\left(\frac{d}{dt}(1+0.12\cos\theta)\right)\right)$$

$$\frac{dr}{dt} = \frac{-4995}{(1+0.12\cos\theta)^2}(0.12\sin\theta)\frac{d\theta}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{(4995)(0.12)\left(\sin\frac{2\pi}{3}\right)}{\left(1+0.12\cos\left(\frac{2\pi}{3}\right)\right)^2}(2.7)\left(\frac{\pi}{180}\right)\text{ rad/min} \approx 27.684$$

25)



$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \left(\frac{1}{3}\pi\right)\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$

$20 = \left(\frac{\pi}{3}\left(2\cdot\frac{5}{12}(16)\right)\cdot\left(\frac{5}{12}\frac{dh}{dt}\right)\cdot(16) + \left(\frac{5}{12}16\right)^2\frac{dh}{dt}\right)$

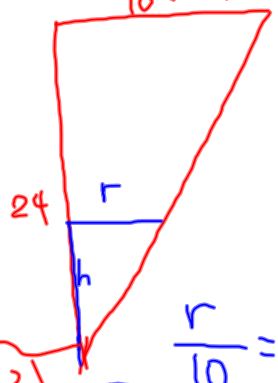
Know

$\frac{dV}{dt} = +20\frac{ft^3}{min}$

instant: $h = 16$ ft

Need to know

$\frac{dh}{dt}$



$\frac{r}{10} = \frac{h}{24}$

$\frac{12}{5}r = h$

$r = \frac{5}{12}h$

@ $h = 16$

$r = \frac{5}{12}\cdot 16$

$20 = \frac{dh}{dt}\left(\frac{\pi}{3}\cdot 2\cdot\frac{5}{12}16\cdot\frac{5}{12}\cdot 16 + \left(\frac{5}{12}\cdot 16\right)^2\right)$

$20 = \frac{dh}{dt}\left(\frac{5}{12}\cdot 16\right)^2\left(\frac{2\pi}{3}+1\right)$

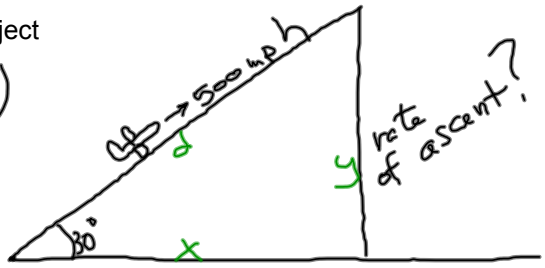
$\frac{12}{5}\frac{dr}{dt} = \frac{dh}{dt}$

$\frac{dr}{dt} = \frac{5}{12}\frac{dh}{dt}$

$\frac{20\cdot 12^2}{16^2\cdot 5^2\left(\frac{3+2\pi}{3}\right)} = \frac{dh}{dt}$

$\approx .145$

29)



$\sin(30°) = \frac{y}{d}$

$\frac{1}{2} = \frac{y}{d} \Rightarrow d = 2y$

Know

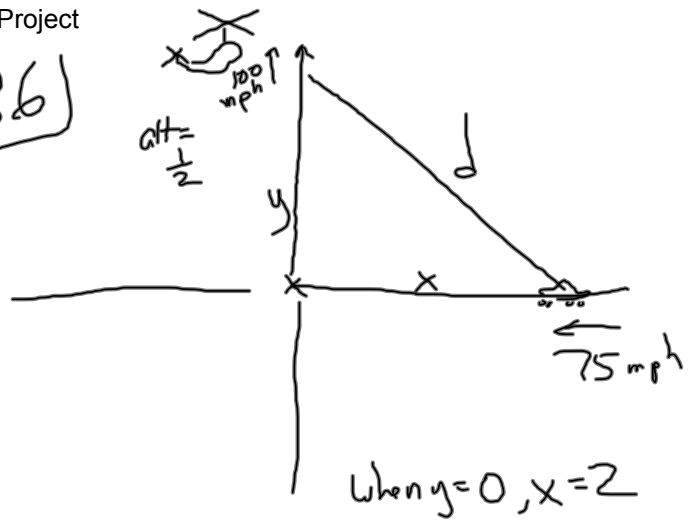
$\theta = 30°$ ALWAYS

$\frac{dd}{dt} = 500 \text{ mph}$

Need to find

$\frac{dy}{dt}$

36)



$h^2 + (\sqrt{x^2+y^2})^2 = d^2$

$h^2 + x^2 + y^2 = d^2$

$\sqrt{h^2+x^2+y^2} = d$

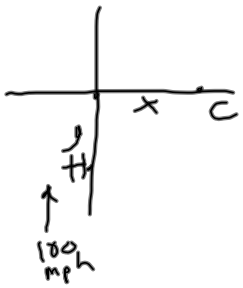
#40, 36, ~~23~~, 44ish, 23, 32

23) see above

36) distance from car → heli.

$d = \sqrt{\left(\frac{1}{2}\right)^2 + x^2 + y^2}$

or $d^2 = \frac{1}{4} + x^2 + y^2$



$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{dy}{dt} = -100 \text{ mph}$ $\frac{dx}{dt} = -75 \text{ mph}$

when $y = 0, x = 2$

$d\frac{dd}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$

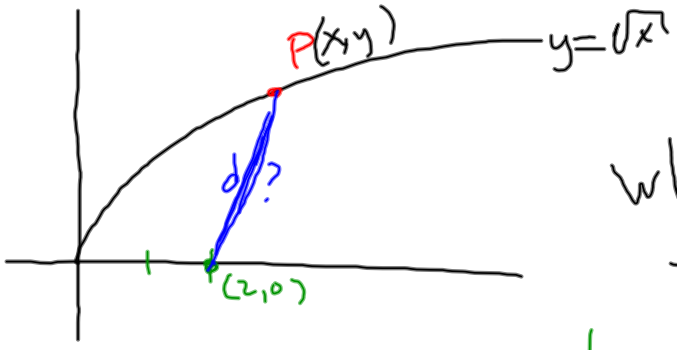
$d\frac{dd}{dt} = 2(-75) + (0)(100)$

$\frac{\sqrt{17}}{2}\frac{dd}{dt} = -150$

$d = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$

$\frac{dd}{dt} = \frac{-300}{\sqrt{17}}$ mi/hr

40)



When $x=3$

$\frac{dx}{dt} = +4$ units/sec

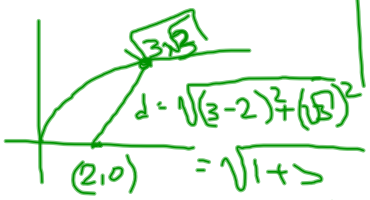
$d^2 = (x-2)^2 + y^2$

$2d\frac{dd}{dt} = 2(x-2)\frac{dx}{dt} + 2y\frac{dy}{dt}$

$d^2 = (x-2)^2 + (\sqrt{x})^2$

$d^2 = (x-2)^2 + x$

$2d\frac{dd}{dt} = 2(x-2)\frac{dx}{dt} + \frac{dx}{dt}$



$d = \sqrt{(3-2)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

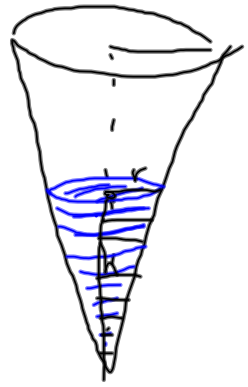
$2(2)\frac{dd}{dt} = 2(3-2)(4) + (4)$

$4\frac{dd}{dt} = 8 + 4 = 12$

$\frac{dd}{dt} = \frac{12}{4} = 3$ units/sec

#40, 36, 23, 44ish

44)



$S_{\text{theta}} = \pi r^2$

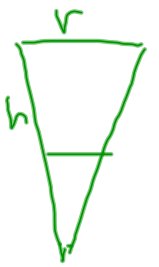
find $\frac{dh}{dt}$

$\frac{dV}{dt} = k\pi r^2$

b/c it's a cone we know

$r = ch$

$V = \frac{1}{3}\pi r^2 h$



$\frac{r}{h} = c$

$V = \frac{1}{3}\pi (ch)^2 h$

$V = \left(\frac{\pi}{3}c^2\right) h^3$

$\frac{dV}{dt} = \left(\frac{\pi c^2}{3}\right)\left(3h^2 \frac{dh}{dt}\right) = (\pi c^2)\, h^2 \frac{dh}{dt}$

$k\pi(ch)^2 = k\pi r^2 = \frac{dV}{dt} = (\pi c^2)\, h^2 \frac{dh}{dt}$

$c^2 k\pi \cdot h^2 = \frac{dV}{dt} = (\pi c^2)\, h^2 \frac{dh}{dt}$

$\frac{c^2 k\pi \cdot h^2}{\pi c^2 h^2} = \frac{dh}{dt}$

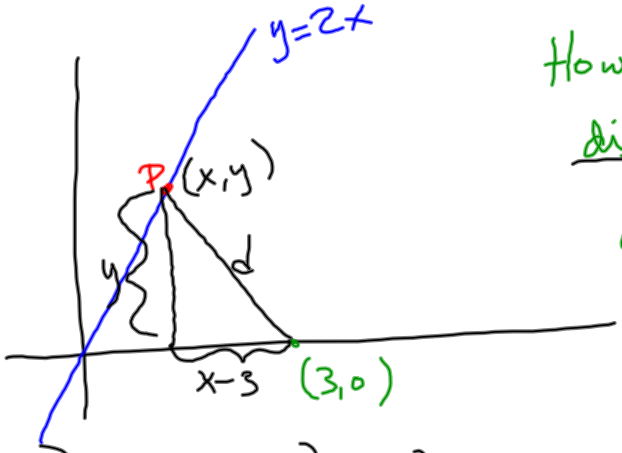
$k = \frac{dh}{dt}$

3.7 Project

2010-11-19 Pd 2

#40, 36, 23, 44ish

39)



How fast is the distance between pt P & (3,0) changing at the instant when P=(3,6)?

$x=3$
$y=6$
$d=6$

$\frac{dx}{dt} = -2$ units/sec

$d^2 = (x-3)^2 + y^2$

$2d\frac{dd}{dt} = 2(x-3)\frac{dx}{dt} + 2y\frac{dy}{dt}$

$d\frac{dd}{dt} = (x-3)\frac{dx}{dt} + y\frac{dy}{dt}$

$6\frac{dd}{dt} = (3-3)(-2) + (6)\frac{dy}{dt}$

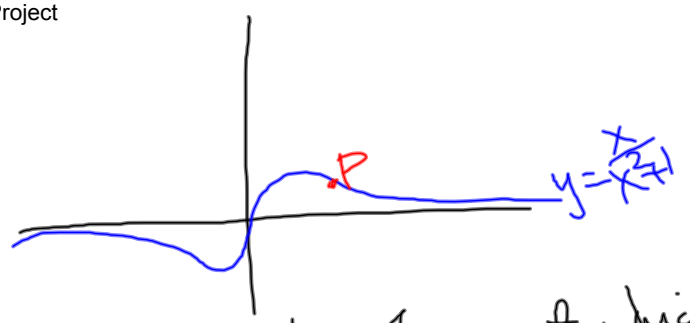
$y = 2x$

$\frac{dy}{dt} = 2\frac{dx}{dt} = 2(-2) = -4$

$6\frac{dd}{dt} = 0 + 6(-4)$

$\frac{dd}{dt} = \frac{-24}{6} = -4$ units/sec

41)



$\frac{dx}{dt}=3\frac{dy}{dt}$

Find all values of x at which the rate of change of x with t is three times that of y.

$y=\frac{x}{x^2+1}$

$\frac{dy}{dt}=\frac{\left(\frac{dx}{dt}\right)(x^2+1)-(x)\left(2x\frac{dx}{dt}\right)}{(x^2+1)^2}$

$\frac{1}{3}\frac{dx}{dt}=\frac{\frac{dx}{dt}(x^2+1)-2x^2\frac{dx}{dt}}{(x^2+1)^2}$

$\frac{1}{3}(x^2+1)^2\frac{dx}{dt}=(-x^2+1)\frac{dx}{dt}$

$\frac{1}{3}(x^4+2x^2+1)\frac{dx}{dt}=(-x^2+1)\frac{dx}{dt}$ or $(x^4+2x^2+1)\frac{dx}{dt}=(-3x^2+3)\frac{dx}{dt}$

$\frac{1}{3}(x^4+5x^2-2)\frac{dx}{dt}=0$

$x^4+5x^2-2=0$

$(x^2)^2+5(x^2)-2=0$

$x^2=\frac{-5\pm\sqrt{5^2-4(1)(-2)}}{2}=\frac{-5\pm\sqrt{33}}{2}$

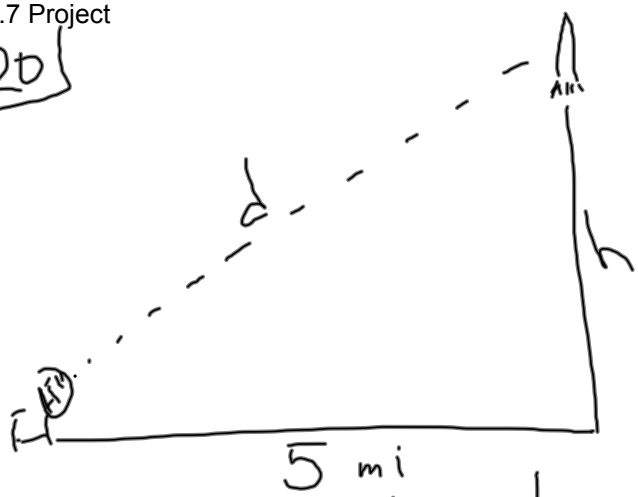
$x^2=\frac{-5-\sqrt{33}}{2}$ or $x^2=\frac{-5+\sqrt{33}}{2}$

not possible : $x=\pm\sqrt{\frac{-5+\sqrt{33}}{2}}$

$(x^4+2x^2+1)\frac{dx}{dt}=(-3x^2+3)\frac{dx}{dt}$

$(x^4+2x^2+1)\frac{dx}{dt}-(-3x^2+3)\frac{dx}{dt}=0$

20)



x^2

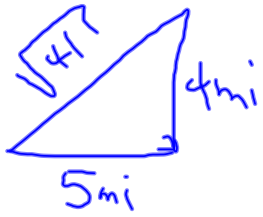
$5^2 + h^2 = d^2$ ✓

$2x\frac{dx}{dt}$ $2h\frac{dh}{dt} = 2d\frac{dd}{dt}$

$h\frac{dh}{dt} = d\frac{dd}{dt}$ ✓

At instant when $h = 4$ mi and $\frac{dd}{dt} = 2000 \frac{mi}{hr}$

what is $\frac{dh}{dt}$

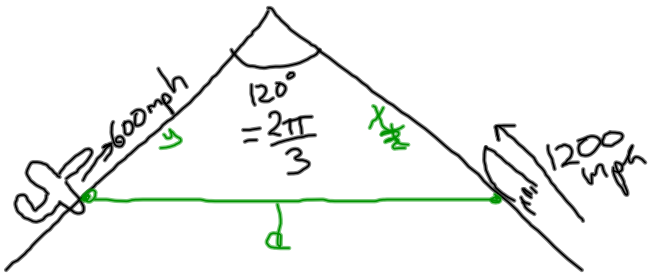


$h\frac{dh}{dt} = d\frac{dd}{dt}$

$4\frac{dh}{dt} = d(2000)$ → $4\frac{dh}{dt} = \sqrt{41} \cdot 2000$

$\frac{dh}{dt} = \frac{2000}{4}\sqrt{41} \frac{mi}{hr}$

35)



$\int e^x\,dx = f(u^n)$

Law of Cosines

$$d^2 = x^2 + y^2 - 2xy\cos\left(\frac{2\pi}{3}\right)$$

$$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} - 2\cos\left(\frac{2\pi}{3}\right)\left[\frac{dx}{dt}y + x\frac{dy}{dt}\right]$$

$$d\frac{dd}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt} - \left(-\frac{1}{2}y\frac{dx}{dt} - \frac{1}{2}x\frac{dy}{dt}\right)$$

$$d\frac{dd}{dt} = \frac{dx}{dt}\left(x + \frac{y}{2}\right) + \frac{dy}{dt}\left(y + \frac{x}{2}\right)$$

$2^2 + 4^2 - 2(2)(4)\left(-\frac{1}{2}\right)$

$= 4 + 16 + 8 = 28$

$c = \sqrt{28} = 2\sqrt{7}$

$$2\sqrt{7}\frac{dd}{dt} = (-1200)(5) + (-600)\left(2 + \frac{4}{2}\right)$$

$$\frac{dd}{dt} = \frac{-6000 - 2400}{2\sqrt{7}} = \frac{-4200}{\sqrt{7}} = -600\sqrt{7}$$ mph

$$-\frac{4200\sqrt{7}}{7}$$

37) $\frac{xy^3}{1+y^2} = \frac{8}{5}$ $(1,2)$ $\frac{dx}{dt} = 6$ units/sec

$$\frac{(xy^3)'(1+y^2) - (xy^3)(1+y^2)'}{(1+y^2)^2}$$

$(xy^3)' =$
$(x')(y^3) + (x)(y^3)'$
$y^3\frac{dx}{dt} + 3y^2x\frac{dy}{dt}$

$$\frac{\left(y^3\frac{dx}{dt} + 3y^2x\frac{dy}{dt}\right)(1+y^2) - (xy^3)(2y\frac{dy}{dt})}{(1+y^2)^2}$$

$x=1$
$y=2$
$\frac{dx}{dt}=6$

$$\frac{\left[2^3(6) + 3(2^2)(1)\frac{dy}{dt}\right](1+2^2) - \left(2^3\left(4\frac{dy}{dt}\right)\right)}{25}$$

$$\frac{\left[48 + 12\frac{dy}{dt}\right](5) - \left(32\frac{dy}{dt}\right) = 0}{25}$$

$$\frac{240}{25} + \frac{60}{25}\frac{dy}{dt} - \frac{32}{25}\frac{dy}{dt} = 0$$

$$\frac{28}{25}\frac{dy}{dt} = -\frac{240}{25}$$

$\frac{dy}{dt} \approx -8.57 = -\frac{60}{7}$ units/sec

$$\frac{-240/25}{28/25}$$

$$\frac{-\frac{240}{25}}{\frac{28}{25}} \cdot \frac{(25)}{(25)} = \frac{-240}{28} = \frac{-12}{14} = -\frac{6}{7}$$